2012

PHYSICS

(Honours)

FIRST PAPER

(Part—I / 2008 Syllabus)

Full Marks : 100

Time : 4 hours

The figures in the margin indicate full marks for the questions

Answer **all** questions

1. Answer the following questions : 1×10=10

(a) Show that $\vec{\nabla}\phi$ is a vector perpendicular to the surface $\phi(x, y, z) = c$, where c is a constant.

(b) Write down the condition for a matrix A to be Hermitian.

(c) Write down the Dirichlet's conditions for a Fourier series.

(d) A disc of mass 200 gm and radius 1 cm is rolling with a velocity 5 cm/sec. Find its rotational kinetic energy.

(e) State the important characteristics of central force.

(f) Write down the difference between gravitational potential and electrostatic potential.

(g) Distinguish between streamline motion and turbulent motion.

(h) If some organic substance is allowed to dissolve into a certain liquid, then what will be the change in surface tension of the liquid?

(i) Write the expressions for the thermodynamic potential at constant volume and thermodynamic potential at constant pressure.

(j) What do you mean by resonance in a mechanical system?

2. (a) (i) State and prove Green's theorem in a plane.

(ii) Verify Gauss's divergence theorem for the vector $\vec{A} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ taken over a cube $0 \le x, y, z \le 1$.

(iii) Find the eigenvalues and eigenvectors of the square matrix

$$\begin{pmatrix} 5 & 2 \\ 2 & 2 \end{pmatrix}$$

(1+4)+5+5=15

(Continued)

Or

(b) (i) State and prove Stokes' theorem.

(ii) Show that

$$\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

(iii) Show that all the eigenvalues of a Hermitian matrix are real.

(iv) For $f(x) = x^2$, $-\pi \le x \le \pi$, expand $f(x)$ in Fourier series and hence show that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

(1+4)+3+3+4=15

3. (a) (i) Derive an expression for the moment of inertia of a thin spherical shell about an axis passing through one of the diameters of the shell. Also find the radius of gyration in this case.

(ii) What do you mean by Coriolis force? Explain the role of this force in formation of cyclone.

(iii) Let us consider the motion of a particle of mass m in a central force field and (r, θ) be the position of the particle at any instant t in plane polar coordinate. Considering $u = \frac{1}{r}$,

(Turn Over)

show that the kinetic energy of the particle at that instant is

$$T = \frac{|L^2|}{2m}\left[\left(\frac{du}{d\theta}\right)^2 + u^2\right]$$

where $\vec{L}$ is the angular momentum of the particle. (5+1)+(1+3)+5=15

Or

(b) *(i)* State and prove Kepler's third law of planetary motion.

(ii) What do you mean by geostationary satellite? Derive an expression for altitude of such a satellite from the surface of earth assuming the orbit of the said satellite as circular.

(iii) Consider a particle moving along a space curve with a velocity $\vec{v}$. Show that its acceleration is given by

$$\vec{a} = \frac{dv}{dt}\hat{T} + \frac{v^2}{\rho}\hat{N}$$

where $\hat{T}$ is the unit vector along the tangent, $\hat{N}$ is the unit vector along the normal and ρ is the radius of curvature. (1+4)+(1+3)+6=15

4. *(a)* *(i)* Describe Kater's reversible pendulum and deduce the necessary working formula for determining acceleration due to gravity using this pendulum.

(ii) Show that in case of torsion of a cylinder, couple per unit twist is

$$C = \frac{\pi \eta r^4}{2l}$$

where l is the length and r is the radius of the cylinder and η is the modulus of rigidity of the material of the cylinder.

(iii) A metallic bar, 30 cm long, 2 cm broad and 0·2 cm thick is clamped at one end and loaded at the other end with a mass of 10 gm. Calculate the depression at a point 20 cm away from the clamped end. (Take, $Y = 1{\cdot}013 \times 10^{11}$ dyne/cm^2, $g = 980$ cm/sec^2) (2+4)+6+3=15

Or

(b) *(i)* Show that the gravitational potential at a point inside the material of a thick hollow sphere is

$$V = \frac{-3GM(a+b)}{2(a^2+ab+b^2)}$$

where a and b are inner and outer radii of the hollow sphere respectively.

(ii) State Gauss's theorem in gravitation and using it, find the gravitational field intensity at an exterior point due to a uniform solid sphere.

(iii) The gravitational potentials within two thin homogeneous spherical shells of same surface density of mass are in the ratio 1 : 2. Calculate the ratio of their radii.

(iv) Explain the term 'internal bending moment' related to bending of a beam.

5+(1+4)+3+2=15

5. (a) (i) What do you mean by Newtonian and non-Newtonian fluid?

(ii) Define surface tension and surface energy of a liquid. Establish a relation between them at an absolute temperature T.

(iii) A capillary tube of radius r and length l is fitted at the bottom of a cylindrical container of cross-section α. Initially, there is a liquid in the container up to height H. What time would be required for the half of the liquid to flow out? The coefficient of viscosity of the liquid is η.

(Continued)

(iv) Calculate the work done on the film in blowing a soap bubble from a diameter of 4 cm to 30 cm if its surface tension be 45 CGS unit.

(1+1)+(1+1+4)+4+3=15

Or

(b) (i) Derive an expression for torque on a cylinder placed in a rotating liquid.

(ii) What is critical velocity? What is the significance of Reynold's number?

(iii) A Pitot tube is fixed on the wing of an aeroplane to measure the speed of the aeroplane. The tube contains a liquid of density 800 kg/m^3. The difference in level between two limbs is 0·5 metre. Density of air is 1·293 kg/m^3. Calculate the speed of the aeroplane.

(iv) An air bubble of radius 1 mm exists in a liquid of surface tension 0·075 N/m and density 1000 kg/m^3 at a depth of 10 cm below the surface of a liquid. By what amount is the pressure inside the bubble greater than the atmospheric pressure? (Take, g as 9·8 m/sec^2)

5+(1+2)+3+4=15

(Turn Over)

6. *(a)* *(i)* What do you mean by isothermal and adiabatic process?

(ii) Write down Clausius statement of second law of thermodynamics.

(iii) Discuss Carnot's cycle by a P-V diagram for an ideal gas. Write expressions for the work done in every step of complete cycle of operation and calculate the efficiency of Carnot's engine.

(iv) Show that the difference between two specific heats

$$C_P - C_V = \frac{-T\left(\frac{\partial V}{\partial T}\right)_P^2}{\left(\frac{\partial V}{\partial P}\right)_T}$$

Hence show that

$$C_P = C_V - \frac{T\gamma^2 V}{\beta_T}$$

where β_T is the isothermal compressibility and γ is the coefficient of thermal expansion of volume. (1+1)+2+(2+5)+(3+1)=15

Or

(b) *(i)* State and prove Kirchhoff's law of radiation.

(ii) Show that the energy density of radiation inside a uniformly heated enclosure is

$$E = \frac{4\pi C}{K}$$

where the symbols have their usual meanings.

(iii) A copper block of 400 gm mass and with total heat capacity at constant pressure of 150 J/deg at 100 °C is placed in a tank of 10 °C. Calculate the change in entropy of the universe. (1+6)+5+3=15

7. *(a)* *(i)* Show that the velocity of longitudinal wave in a gaseous medium is

$$v = \sqrt{\frac{E}{\rho}}$$

where E = bulk modulus of the medium and ρ = density of the medium.

(ii) Find the resultant of two simple harmonic motions acting at right angles to each other having same time periods but different amplitudes when the phase difference between two vibrations are (1) $\frac{\pi}{4}$, (2) $\frac{\pi}{2}$ and (3) π.

(iii) How are the combinational tones explained by Helmholtz intensity theory? 5+6+4=15

Or

(b) (i) What is Doppler effect? Deduce an expression for change in frequency due to this effect when both the source and listener are in motion.

(ii) A string of length l fixed at both ends at $x = 0$ and $x = l$ is struck with a hammer at $x = \frac{3l}{4}$. Find the displacement at any point x as a function of time. Show that Young's law is also obeyed by the vibrating struck string.

(iii) Distinguish between bel and phon. (1+4)+(6+2)+2=15

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